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Critical review of guidelines for checking vibration serviceability of post-tensioned concrete floors

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Abstract

As spans and slenderness of post-tensioned concrete office floors increase, their vibration behaviour is becoming increasingly important. Although now recognised worldwide to be an important design issue, detailed guidance on the checking of the vibration serviceability of post-tensioned floors is not readily available in national building codes of practice. The only detailed guidelines developed worldwide specifically for checking vibration serviceability of post-tensioned office floors have been published by the UK Concrete Society in 1994. These guidelines are generic, could be and are being used in countries where specific national guidance is lacking. However, the Concrete Society guidelines have been found to be problematic in each of the three key aspects of floor vibration serviceability assessment. First, in the modelling of walking excitation, an unrepresentative walking forcing function is assumed. Second, as the guidelines were not experimentally verified, some unwarranted assumptions and simplifications are made in the modelling of the floor structure, in particular its boundary conditions. Finally, the method of calculation of the floor responses is somewhat simplistic. Here, the possibility that modes of vibration higher than the fundamental may be excited in resonance by walking is neglected.

Keywords: Concrete floors; Prestressed concrete; Vibration; Serviceability; Guidelines; Critical review

1. Introduction

A floor is an integral part of practically every modern industrial, commercial or residential building. Advanced floor construction techniques, such as post-tensioned concrete slabs, are gaining popularity. Typical applications where post-tensioned floors in particular are competing with other types of floors are offices, car parks, shopping malls, hospitals, apartments and industrial buildings [1].

One of the strong attractions of post-tensioned slab design is the ability to reduce the structural depth to the absolute minimum. This is done by applying more prestress to enhance strength and control deflections. The thinnest possible floors are frequently desired to reduce the overall building height to meet planning restrictions, to reduce the amount of excavation in underground construction or to provide greater space for services within a fixed building envelope. All these benefits may lead to cheaper floor and/or building construction.

However, as the demands and expectations of building users rise, so the performance of floor structures in day-to-day service is becoming increasingly important. Floors generate the second most frequent source of complaints from building users; second only to roofs [2]. Floor performance which is sufficiently poor as to cause users to complain may incur loss of confidence, costly remedial measures and/or litigation [3].

1.1. The floor vibration serviceability problem

To ensure a floor has satisfactory vibration performance is becoming a major design and research challenge. There are several reasons for this, not least the trend towards increased floor spans and slenderness, and reduced floor damping. There is also an increased industrial awareness of the potential of vibration serviceability problems occurring in the development of slender floors in relatively quiet occupancies. However, there is a lack of reliable information as to how to tackle it in the

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design stage. Typically, internationally recognised and widely used guidelines specialised in post-tensioned floors [4–7], either do not mention the problem at all, or, at best, give a general advice that the floor should have satisfactory vibration performance without being specific as to how to achieve this.

Perceptible vibrations annoying the floor occupants and dysfunctional equipment sensitive to vibrations are manifestations of the lack of floor vibration serviceability. In the past, a number of composite steel-concrete and timber floors were found to be too lively under normal everyday dynamic excitations, although they were sufficiently strong and did not deflect excessively. Following complaints by their users, the floors were tested and often failed the vibration serviceability design criterion [8,9] which has emerged as a general design requirement in practically all modern building codes based on limit state design principles.

Unlike composite steel-concrete and timber floors, monolithic cast in situ concrete floors, which are used widely in office construction, are heavier and have an excellent track record with regard to their past vibration performance [10,11]. Indeed, complaints about their vibration behaviour are rare, in fact, almost non-existent. Therefore, the focus of researchers in the past has been mainly on the lighter and more lively composite steel–concrete and timber floors. However, an ongoing push to utilising prestressed post-tensioned floors for longer more slender spans could result in similar problems of excessive liveliness.

Designers of post-tensioned concrete floors worldwide are currently aware of the potential problem if relatively long post-tensioned floor spans are required. However, it has also been widely recognised that checking and/or assessing the vibration serviceability of post-tensioned floors is, at the moment, far from being a routine design procedure.

1.2. The Concrete Society vibration serviceability guidelines

The only known attempt to address this issue in post-tensioned floors was made by the UK Concrete Society in their Technical Report 43 (CSTR43) [1] published in 1994. This report is the third and the latest edition of a Concrete Society handbook which describes the state of the art in the design of post-tensioned floors. The handbook has been used since the late-1970s throughout the world. The previous editions were Technical Report 17 [12] and Technical Report 25 [13]. The 1994 edition of the handbook contains, for the first time, procedures for checking the vibration serviceability of long-span post-tensioned floors supporting office environments. In principle, these guidelines are generic, could be and are being used in countries where specific national guidance is lacking.

Unfortunately, some initial trials [14,15], and the experience which the authors gathered through their own work and from other colleagues in industry, indicate that the vibration serviceability provisions in CSTR43 are producing erratic results and, frequently, over-conservative designs. In 1994, Williams and Waldron [15] attempted to verify some elements of the CSTR43 provisions against fundamental frequencies and mode shapes measured on a number of prototype post-tensioned floors. They concluded that the guidelines were "unreliable" without going into the reasons. Feedback from industry confirms this, as back analyses of many older designs, which had been built and are in service for years without adverse comment, showed that some of these also failed when the new CSTR43 guidelines were applied.

At the time of the publication of the CSTR43 guidelines, very little information on dynamic performance of as-built post-tensioned floors existed. Therefore, the guidelines were not validated against any experimental data gathered on as-built full-scale floors [16]. However, since then, new experimentally verified data on the performance of the guidelines have been gathered and this is further enhanced by practical experience gained with the application of the guidelines. Therefore, in the light of this new information, this paper presents an evaluation of the guidelines and assumptions on which they were developed.

Without information on exactly why the CSTR43 guidelines are seemingly unreliable it is difficult to improve on them. The principal aim of this paper is therefore to outline the likely reasons for the unsatisfactory performance of CSTR43 vibration serviceability provision. However, to do this it is important to understand first the overall philosophy and background of modern procedures for vibration serviceability checks employed throughout the world (and in CSTR43).

2. Current state-of-the-art of vibration serviceability

Although the problem of floor and human vibration is, reportedly, very difficult to deal with [17], design decisions have to be made in contemporary design practice where the vibration serviceability limit state has to be considered. A state-of-the-art framework for providing a general procedure for checking floor vibration serviceability is given by the International Standardisation Organisation (ISO) [18]. The first step towards the assessment of vibration serviceability of floors is to identify and characterise the following three key factors: (i) the vibration source, (ii) the transmission path, and (iii) the receiver.

2.1. Vibration source

The sources of floor vibration generate dynamic actions which may vary both in time and in space. They can be divided into two groups, namely inside (or internally generated), and outside (or externally generated).

Typical examples of internal floor vibration sources, defined as originating in the building, are:
- human excitation, such as walking, running and jumping;
- machinery, such as mechanical and electrical plants, elevators, lift trucks, punches and presses;
- various construction activities within the building.

ISO [18] also defines two general classes of vibration serviceability problems depending on the nature of the vibration source. Class A problems involve vibration sources which vary both in space and in time, for example walking occupants. These are much more complex than Class B problems, which are caused by stationary vibration excitation which changes only in time, such as that caused by mounted machinery. As regards the Class A problems, ISO 10137 guideline [18] states that:

> The complexity of these problems is one reason why many of them have been treated by empirical methods, or by extensive use of measurements on similar existing structures.

It should be noted here that the CSTR43 guidelines are concerned only with the performance of long-span office floors under human-induced excitation. Thus, this is an example of a Class A problem where the main source of excitation is assumed to be a single person walking across the floor.

2.2. Transmission path

The transmission path is a medium which passes on excitation from the vibration source to the receiver. Structural components in buildings transmitting vibrations could be foundations, columns, walls and floors, whereas non-structural paths may be raised access floors, removable partitions, cladding etc. Physical properties of the transmission path, such as stiffness, mass or damping, modify the vibration excitation at the source into a structural response felt at the position of the receiver. The key dynamic properties of the vibration path are natural frequencies, mode shapes and modal damping ratios. As the floor vibration serviceability problem is typically linear, these three properties are sufficient to calculate the floor vibration response felt by the receiver of the vibrations.

2.3. Receiver

ISO 10137 defines the receiver as "the object or person for which the vibration effects are to be assessed". The persons are, obviously, the human occupants of the building, whereas the objects can be either vibrating structural or non-structural elements (windows, walls, beams, slabs etc.) or contents of the building such as instruments or machinery. The amount of vibration passed to the receiver should be evaluated in accordance with certain established criteria. This evaluation is the core problem of the vibration serviceability assessment.

For floors, two types of vibration serviceability assessment exist: (1) evaluation by calculation during the floor design stage; and (2) evaluation by vibration measurement of already built full-scale floor structures [17,18].

Although this rationalisation of the vibration serviceability problem into the characterisation of the vibration source, transmission path and receiver may seem simple, it is actually a very difficult task. It requires a thorough appreciation of the floor vibration phenomenon in order to comprehend the appropriate vibration serviceability criteria and to be able to apply the latest national and international vibration evaluation standards. These standards are far from perfect, but Griffin [17] justifies this by saying:

> The shaking of the human body – a complex, active, intelligent, dynamic structure should not be expected to have a single, simple or easily predictable consequence.

2.4. RMS vs VDV assessment

There are two parameters which are typically used in modern codes of practice for assessing the amount of vibration and its effects on the human occupants of office floors [18–20]. These are root-mean-square (RMS) accelerations, and the more recently established [17] so called '4th power' methods, such as the root-mean-quad and vibration dose value (VDV) methods.

Vibrations in buildings are seldom simple sinusoids. Often, the vibration time signatures are modulated, transient or random, and contain a range of frequencies. To take into account the varying human susceptibility to vibrations at different frequencies, the time histories must be 'weighted' [17]. After being weighted, the most common method for mapping such vibrations into a single numerical (or effective) value, to be compared with a vi calculate the RMS of the weighted acceleration time-history $a_w(t)$ using the following formula:

$$\text{RMS acceleration} = \left[\frac{\int_{t_1}^{t_2} a_w^2(t)\,dt}{t_2 - t_1}\right]^{1/2}. \qquad (1)$$

RMS acceleration is used as it is a measure of the total vibration causing distress to the human body. Greater RMS accelerations correspond to higher vibration magnitudes causing more annoyance. However, an assessment of human distress using the RMS relationship is only appropriate for, as Griffin defines them, "well behaved" vibrations which are steady-state long-duration periodic or stationary random. If the vibrations are relatively short-lived transients, as may be the case with those caused by human walking, then the RMS acceleration no longer appears to be a reliable effective value.

A method that addresses this problem and is gaining acceptance internationally is based on the vibration dose value. This method is suitable for assessing all types of vibratory motion (periodic, random and transient). The VDV is a cumulative measure of the vibration transmitted to a human receiver during a certain period of interest $T = t_2 - t_1$, and is defined as

$$\text{VDV} = \left[\int_{t_1}^{t_2} a_w^4(t)\,dt\right]^{1/4}. \qquad (2)$$

Although the VDV is, seemingly, a more reliable parameter which has been shown to correlate relatively well with human sensation of all types of vibration in numerous recent experimental studies [17], it does not have a physical meaning. Whereas units for RMS accelerations are ms^{-2} the VDV units are $ms^{-1.75}$.

The relevant international standard for vibration serviceability of building floors is Part 2 of ISO 2631 [19] entitled Evaluation of human exposure to whole body vibration: Continuous and shock vibration in buildings (1–80 Hz).

3. Description of the CSTR43 method

The CSTR43 vibration serviceability provision is developed specifically for in situ concrete floors. It integrates considerations of the floor vibration source, path and assessment, and presents them as one package. This is a very popular approach adopted by a number of floor vibration design guidelines worldwide, including the SCI/CIRIA [21] *Design guide on the vibration of floors*. However, the SCI/CIRIA guide is developed for composite steel-concrete floors and, therefore, it is not directly applicable to in situ post-tensioned floors considered in CSTR43.

Appendix G of CSTR43 proposes a method for calculating the dynamic response of post-tensioned floors of various structural configurations. Considering only the information given in Appendix G of CSTR43, the background of the method is not clear. However, the background of the method is a crucial piece of information often needed by practitioners and necessary to perform an informed review of the method. Therefore, based on the information readily obtained from the developers of the CSTR43 method [16], its key assumptions will be explained here.

3.1. Modelling of human-induced excitation

The walking force model determined by Ohlsson (known as M2) [22] is assumed to be the only excitation of the floor. The force shape is assumed to be perfectly periodic and as such, the force is presented in terms of Fourier components given in Table 1.

The M2 walking forcing function corresponds to a pacing frequency of 1.57 Hz (i.e., 1.57 steps/s or 94 steps/min). The data in Table 1 show that the higher harmonics have comparatively lower amplitudes, so only

Table 1
Fourier components of M2 walking (static weight = 687 N)

Harmonic no.	Frequency f_h (Hz)	Amplitude P_h (N)	Phase φ_h (°)	Dynamic loading factor
1	1.57	124	16.4	124/687 = 0.180
2	2 × 1.57 = 3.14	93	199.0	93/687 = 0.135
3	3 × 1.57 = 4.71	38	63.3	38/687 = 0.055
4	4 × 1.57 = 6.27	27	63.2	27/687 = 0.039
5	5 × 1.57 = 7.84	13	350.9	13/687 = 0.019
6	6 × 1.57 = 9.41	10	17.8	10/687 = 0.015
7	7 × 1.57 = 10.98	12	3.15	12/687 = 0.017
8	8 × 1.57 = 12.55	11	340.6	11/687 = 0.016
9	9 × 1.57 = 14.12	8	318.6	8/687 = 0.012
10	10 × 1.57 = 15.69	4	329.8	4/687 = 0.006
11	11 × 1.57 = 17.25	5	324.5	5/687 = 0.007
12	12 × 1.57 = 18.82	4	306.6	4/687 = 0.006
13	13 × 1.57 = 20.40	4	284.3	4/687 = 0.006
14	14 × 1.57 = 21.96	4	257.3	4/687 = 0.006
15	15 × 1.57 = 23.53	5	234.4	5/687 = 0.007

the first 13 harmonics were considered as important. Having only this excitation available, and in order to excite at least one resonant frequency of the floor, it was further assumed that one of the first 13 harmonics always excites the floor's fundamental mode(s). Therefore, it was necessary to calculate all possible modes of vibration of the floor between 1.57 and 20.4 Hz. This led to another series of assumptions related to the mathematical modelling and analysis of the floor.

3.2. Modelling of the floor structure and response calculations

The calculation of natural frequencies and dynamic response in CSTR43 is based on a simplified 'rectangular plate method' coupled with a somewhat streamlined mode superposition method.

3.2.1. Calculation of natural frequencies and mode shapes

The rectangular plate method can be used for calculation of dynamic responses of floors with a regular rectangular grid (Fig. 1(a)). The method considers separately two typical bays running in both orthogonal directions (shaded areas in Fig. 1(a)). It assumes that each bay can be considered as a rectangular plate simply supported along all four edges. If the bending stiffness in two orthogonal directions is different then each of the plates should be considered as orthotropic. The modes of vibration of such plates are assumed to be the same as the modes of vibration of the whole floor. As the analysis is concerned only with the lowest modes of vibration, the ordinates of the global mode shape of the floor considered must alternate between adjacent bays (i.e. rectangular plates) in both orthogonal directions. This is how two families (x and y) of mode shapes are created (Fig. 1(c)).

If, say, the x-direction bay is considered (Fig. 1(b)), then the mode shapes and frequencies of the simplified rectangular plate are described by the numbers of half-waves in each of the two directions. In this case the natural frequency corresponding to such an assumed mode shape pattern is calculated from a well-known formula [23] as

$$f = \frac{\pi}{2}\sqrt{\frac{n_a^4}{(n_x l_x)^4} EI_x + \frac{n_b^4}{(n_y l_y)^4} EI_y} \cdot \sqrt{\frac{1}{m}}, \tag{3}$$

where n_a is the number of half-waves in the x-direction, n_b is the number of half-waves in the y-direction, EI_x and EI_y are the bending stiffnesses per unit width in the x- and y-directions, respectively, and m is the slab's mass per unit area.

CSTR43 assumes that m is uniformly distributed and consists of the mass corresponding to the self weight of all permanent structural and non-structural elements and installations plus 10% of the mass due to imposed gravity load. Also, if the floor is ribbed, i.e., orthotropic, then it is further assumed that beams are narrow so that only the thickness of the slab contributes to the lateral stiffness of the plate in the direction perpendicular to the ribs. It should be mentioned that this may cause large differences between EI_x and EI_y leading to numerous and closely spaced natural frequencies when n_b (or n_a), is kept constant whilst n_a (or n_b), is changed (depending on which direction is analysed).

As the CSTR43 method is concerned only with the low-frequency modes, only one half-wave is considered in the stiffer span direction of the bay analysed. For example, in the case shown in Fig. 1(b), and considering Eq. (3), the shorter span is considered to be stiffer, so $n_a = 3$ and $n_b = 1$. CSTR43 considers modes with two half-waves in the stiffer direction as generally having much higher frequencies and not excitable by walking.

3.2.2. Calculation of acceleration response

For the given floor configuration simplified by equivalent rectangular plates, Eq. (3) typically produces a number of frequencies between 1.57 and 20.4 Hz excitable by M2 walking. By using a mode superposition technique, the total floor response to all 13 walking harmonics is calculated as a linear superposition of individual responses of each mode to the 13 harmonics of excitation. Therefore, the problem has been reduced to the analysis of individual modes each of which can be analysed as an individual mass-spring-damper single-degree-of-freedom (SDOF) system [24].

The acceleration response of a SDOF system having mass m_n to each of the 13 sinusoidal harmonics given in Table 1 is calculated using another well-known formula [24]

$$\ddot{x}_h(t) = \frac{-(P_h/m_n)(f_h/f_n)}{\sqrt{\left(1-(f_h/f_n)^2\right)^2 + (2\varsigma_n(f_h/f_n))^2}} \sin(2\pi f_h t - \varphi_h), \tag{4}$$

where P_h, f_h and φ_h are the amplitude, frequency, and phase angle of the hth excitation harmonic, as given in Table 1, and m_n, f_n and ς_n are the SDOF mass, natural frequency and damping ratio.

The total SDOF response to all 13 harmonics is given in terms of RMS accelerations, and, as such, has been calculated by summing individual sinusoidal peak acceleration responses $\ddot{x}_{\mathrm{h,peak}}$ to all 13 harmonics (each calculated using Eq. (4)) as follows:

$$\ddot{x}_{\mathrm{RMS}} = \sqrt{\frac{\sum_h (\ddot{x}_{h,\mathrm{peak}})^2}{2}}. \tag{5}$$

These analyses were performed for a range of SDOF systems having constant mass m_n of 1000 kg when the

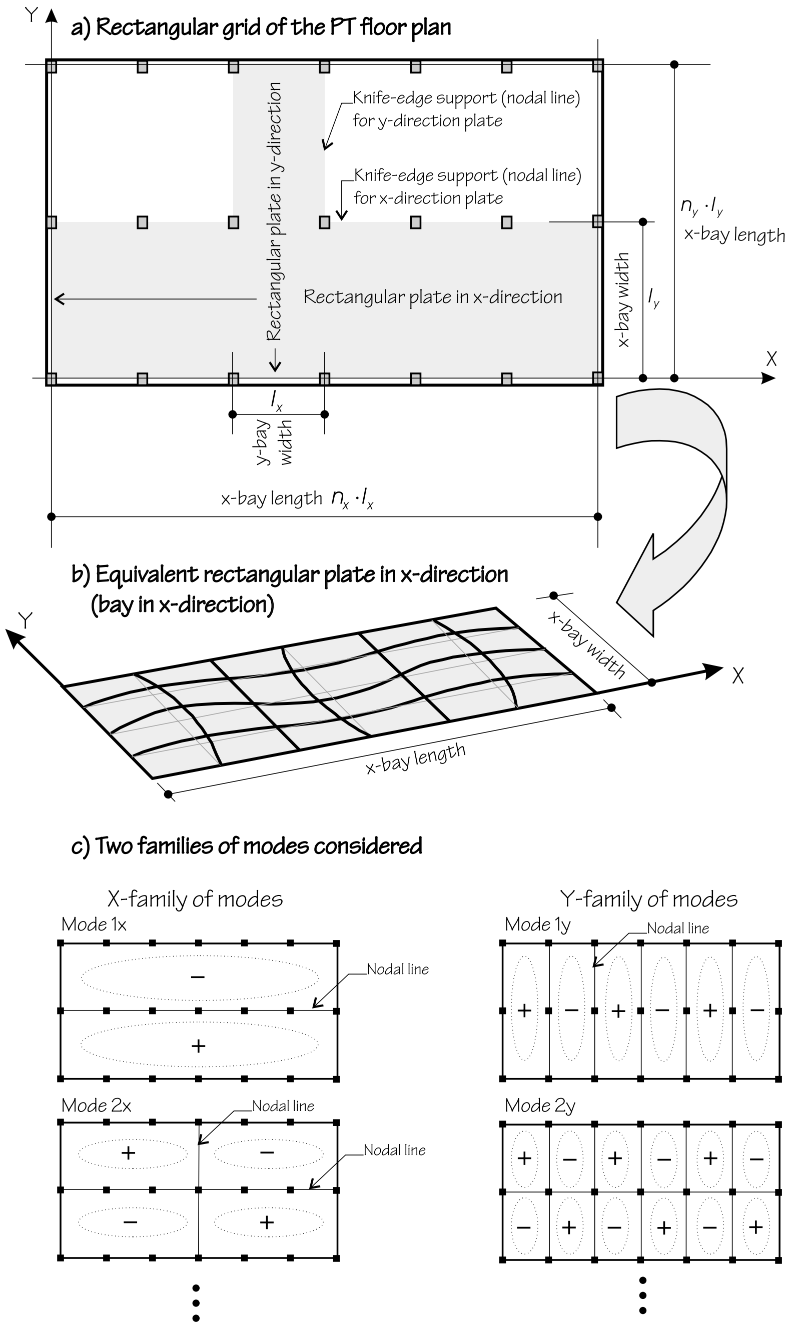

Fig. 1. Rectangular plate method used in CSTR43.

natural frequency f_n of the SDOF system was parametrically increased from 1 to 20 Hz. The RMS response was calculated in terms of a response factor R given as

$$R = \frac{\ddot{x}_{\mathrm{RMS}}}{0.005\ \mathrm{ms}^{-2}}. \tag{6}$$

In other words, the response factor R shows how many times the calculated RMS accelerations are greater (or smaller) than 0.005 $\mathrm{m/s^2}$ which is the RMS acceleration level corresponding to the threshold of feeling vibrations represented as Base Line 1 in ISO 2631-2 [20]. An example of such a calculation is shown in Fig. 2 for a series of SDOF systems when a 2% damping ratio is assumed for all of them. Similar analyses were repeated for four additional series of SDOF systems having 3%, 4%, 6% and 8% critical damping. Naturally, the series of 1000 kg SDOF systems with the smallest damping ratio of 2% had the greatest responses which were amplified due to resources at frequencies corresponding to the M2

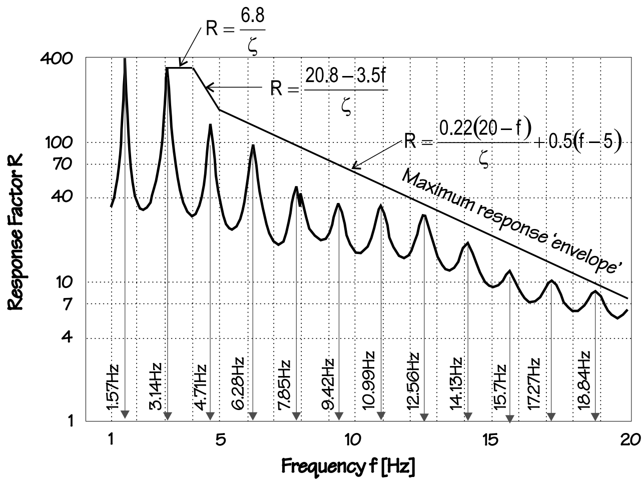

Fig. 2. CSTR43-Response of a SDOF system having mass of 1000 kg to 13 harmonics of walking, when the natural frequency of the system is parametrically changed from 1 to 20 Hz.

walking harmonics (1.57, 3.14 Hz, etc.), as shown in Fig. 2.

These responses served to formulate an envelope (Fig. 2) of maximum response factors R for a SDOF system having 1000 kg mass and varying natural frequencies. These maximum response factors are given as a function of frequency in five frequency regions, as presented in Table 2.

Using the formulae given in Table 2 is conservative and provides an upper bound RMS acceleration response which is higher than the real response of a SDOF system in resonance (Fig. 2).

Another key feature in the development of the CSTR43 vibration serviceability guidelines is the assumption that the modal mass of each floor mode excited is exactly 25% of the total floor mass which is vibrating. This assumption stems directly from the dynamic modelling of the floor bays using simply supported rectangular plates. All modes of such plates have identical modal mass which is exactly one-quarter of the plate's mass when unity scaled mode shapes are used in the calculation of modal masses [25]. This is a well-known feature of simply-supported rectangular plates and one of the key reasons why such plates have to be used when simplifying the calculation of the floor's dynamic response.

Table 2
Maximum response factors R for a 1000 kg SDOF system

Ranges of SDOF natural frequencies	Maximum response factor R
$f_r \leqslant 3$ Hz	$\frac{61.2}{f_r^2 \varsigma}$
$3 \leqslant f_r \leqslant 4$ Hz	$\frac{6.8}{\varsigma}$
$4 \leqslant f_r \leqslant 5$ Hz	$\frac{20.8 - 3.5 f_r}{\varsigma}$
$5 \leqslant f_r \leqslant 20$ Hz	$\left[\frac{0.22 \cdot (20 - f_r)}{\varsigma} + 0.5(f_r - 5)\right]$
$20\ \text{Hz} \leqslant f_r$	7.5

Therefore, CSTR43 proposes that the response factor for the whole floor under a single pedestrian excitation, considering only modes in the r-direction (r is x or y), be calculated as

$$R_r = \frac{R \cdot N_r}{(m n_x n_y l_x)/4} = \frac{C_r N_r}{m n_x n_y l_x l_y}, \tag{7}$$

where $m n_x n_y l_x l_y$ is the mass of the whole floor, and $C_r = 4R$ (Table 3) where R is the maximum response corresponding to a 1000 kg SDOF system (Table 2). As Eq. (4) suggests, the acceleration steady state response of each SDOF system (i.e., mode) is inversely proportional to its mass. Therefore, to obtain a realistic response of each floor mode excited by walking, the 1-tonne maximum response factor R is divided in Eq. (7) by the floor's modal mass expressed in tonnes, which is one-quarter of the total floor mass.

It is important to note here that CSTR43 offers a set of formulae for determining the fundamental frequencies of the floor f_r for both orthogonal directions depending on whether the floor is a solid, coffered or ribbed slab. Such calculated frequencies should be used in conjunction with the formulae given in Table 3.

Finally, the factor N_r in Eq. (7) was empirically developed and it presents a number of floor modes excited by M2 walking. It is given as

Table 3
Maximum response factors C_r the whole floor, as published in CSTR43

Ranges of floor fundamental frequencies	Maximum response factor C_r
$f_r \leqslant 3$ Hz	$4 \cdot \frac{61.2}{f_r^2 \varsigma} = \frac{244.8}{f_r^2 \varsigma}$
$3 \leqslant f_r \leqslant 4$ Hz	$4 \cdot \frac{6.8}{\varsigma} = \frac{27.2}{\varsigma}$
$4 \leqslant f_r \leqslant 5$ Hz	$4 \cdot \frac{20.8 - 3.5 f_r}{\varsigma} = \frac{83.2 - 14 f_r}{\varsigma}$
$5 \leqslant f_r \leqslant 20$ Hz	$4 \cdot \left[\frac{0.22 \cdot (20 - f_r)}{\varsigma} + 0.5(f_r - 5)\right]$ $= \frac{0.88 \cdot (20 - f_r)}{\varsigma} + 2 \cdot (f_r - 5)$
$20\ \text{Hz} \leqslant f_r$	30

$$N_r = 1 + (0.5 + 0.1 \ln \varsigma) \cdot \lambda_r \quad \text{for solid or coffered slabs,} \tag{8}$$

or

$$N_r = 1 + (0.65 + 0.1 \ln \varsigma) \cdot \lambda_r \quad \text{for ribbed slabs,} \tag{9}$$

where λ_r is referred to as the "effective aspect ratio" in CSTR43. This ratio is defined for both orthogonal directions as

$$\lambda_x = \frac{n_x l_x}{l_y} \sqrt[4]{\frac{EI_y}{EI_x}}, \quad \text{for bays in the } x\text{-direction} \tag{10}$$

and

$$\lambda_y = \frac{n_y l_y}{l_x} \sqrt[4]{\frac{EI_x}{EI_y}}, \quad \text{for bays in the } y\text{-direction.} \tag{11}$$

The factor N_r is generally greater for ribbed slabs, and it increases as the damping and effective aspect ratios increase. This parameter is therefore greater for floors with a strong orthotropy and close modes leading to an increased number of modes in the frequency range of interest between 1.57 and 20.4 Hz.

However, it must be stressed here that the factor N_r does not in fact take into account the possibility that a mode closely spaced to the fundamental mode may occur and be excited in resonance. This is because the underlying assumption made while developing simplified formulae for N_r has been that only the fundamental mode of vibration is excited in resonance by one of the walking harmonics. The higher harmonics of the M2 walking model can only by chance excite some of the higher modes and those are not closely spaced to the fundamental mode as the frequency separation is at least 1.57 Hz. This separation is considerable for post-tensioned floors which are typically low frequency floors [21] having fundamental modes between 4 and 7 Hz. Therefore, if the second floor mode, even when predicted by the simplified rectangular plate model, is close to the fundamental, it will not be excited in the CSTR43 method. In essence, the CSTR43 method is an enhanced SDOF method where the majority of the floor response is controlled only by the response of its fundamental mode.

Another underlying assumption in the development of the CSTR43 guidelines is that the excitation force is always applied and the response evaluated at the antinode point (maximum displacement amplitude) of the mode considered. This means that the worst case scenario is assumed as the walking force is treated as a stationary dynamic force exciting the floor at its most susceptible point whereas the response is evaluated at the same point leading to maximum response.

The last important assumption made in the CSTR43 response calculations is that the total floor response factor is the sum of response factors corresponding to two orthogonal directions (Fig. 1(c)):

$$R_{\text{total}} = R_x + R_y. \tag{12}$$

Floor response factors R_x and R_y are determined by repeating virtually the same rectangular plate analysis twice for representative floor stiffnesses and spans in the x- and y-directions.

4. Assessment of the CSTR43 method

An assessment of the CSTR43 method was performed by comparing its results with the experimentally measured modal and response properties on a number of full-scale floors of varying complexity [15,26]. This methodology identified a number of possibly serious shortcomings in the CSTR43 guidelines. These are outlined in the following sub-sections and are illustrated with examples, where appropriate.

Problems were identified in each of the three key aspects of the floor vibration serviceability assessment: (1) in the modelling of the walking excitation, (2) in the modelling of the floor structure, and (3) in the calculation of vibration responses.

4.1. Problems in the modelling of the vibration source

The M2 pedestrian forcing function [22] used in CSTR43, corresponds to a relatively slow pacing rate of only 1.57 steps/s. No allowance was made to incorporate different amplitudes of the fundamental and higher harmonics if the pacing frequency is increased up to 2 Hz or more, which is realistic and, in fact, quite normal [9,27]

Also, when applying this forcing function, an assumption was made that only the fundamental mode is excited in resonance by one of the harmonics between 1.57 and 20.4 Hz. Therefore, close modes of vibration higher than the fundamental are assumed not to be excitable in resonance. This assumption may not be valid as recently demonstrated by response measurements on a number of floors [26,28] where the excitation of higher modes having lower modal masses than the fundamental mode produced the maximum responses.

4.2. Problems in the modelling of floor boundary conditions

One of the main problems with the CSTR43 methodology is its inability to predict the floor's natural frequencies to a sufficient degree of accuracy. Assumptions of simply supported floor edges and of columns acting as pin supports with no rotation restraint, underestimate the natural frequencies. In the analysis performed in accordance with CSTR43 this exposes the floor to much larger amplitudes corresponding to the lower harmonics of walking excitation.

To illustrate this point, Fig. 3 shows a plan and relevant cross sections of an existing high-quality office floor which failed comprehensively the vibration serviceability check proposed in CSTR43. The reason for this failure is shown in Fig. 4 which compares the analytically predicted (by the CSTR43 method) and measured floor responses due to single person walking. Curves 4 and 8 in Fig. 4 correspond to the maximum allowable RMS accelerations in the cases of an 'office' and 'workshop' environments, respectively and as specified in ISO 2631-2. The response measurements were made with the help of a beeping metronome to control pacing rates in order to excite as close as possible the first two modes of vibration established experimentally at 6.4 and 6.9 Hz. Details of the response test procedure are described in detail elsewhere [29] and will not be repeated here, but represented the worst possible floor excitation conditions by a single pedestrian of average weight (80 kg).

It is clear from Fig. 4 that the response predicted by CSTR43 was significantly higher than that determined from full scale testing. Moreover, the fundamental frequency predicted by CSTR43 was only 2.7 Hz whereas the measured value was 6.4 Hz. The main reason for such a large underestimation of the fundamental frequency was the general inability of the CSTR43 guidelines to model the stiffening effects of the in situ cast concrete floor columns and the 'core' area [26] (Fig. 3). The consequence of this is that the office floor designed for a quiet office environment is predicted to fail comprehensively (maximum predicted RMS accelerations above Curve 4 in Fig. 4) but actually performs quite well in real life (maximum measured RMS accelerations well below Curve 4). A practical verification of the measured performance is that no complaints about the liveliness of the floor have been received from its users.

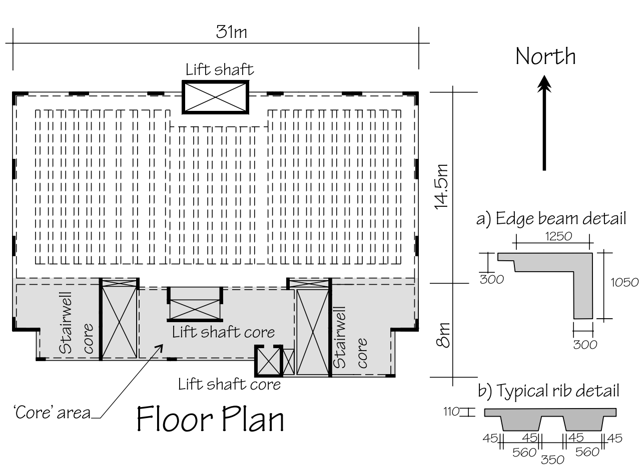

Fig. 3. Layout of in-situ cast post-tensioned concrete floor which initially failed the CSTR43 vibration serviceability check.

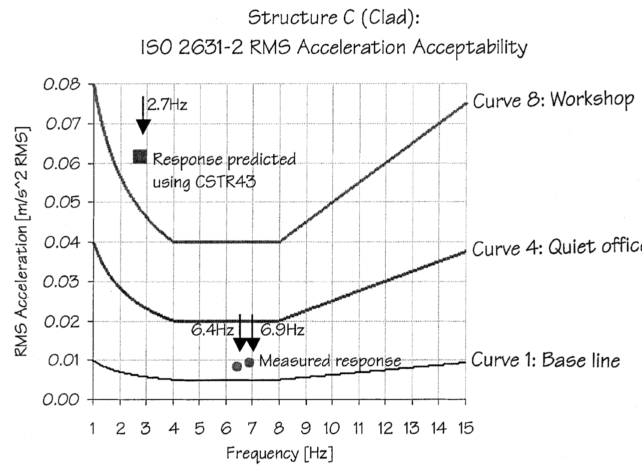

Fig. 4. Vibration serviceability assessment based on CSTR43 provision and measured response from a single person walking.

4.3. Problems in the modelling of column lines

The rectangular plate method which served as the basis for the development of the guidelines in CSTR43 inherently assumes that nodal lines exist along the column lines irrespective of whether beams exist at these positions or not. As the pin-support assumption unduly reduces the floor stiffness and natural frequencies, so the assumption about the nodal lines along the column lines stiffens the floor and increases the natural frequency. Therefore, the effects of the two simplifications tend to cancel each other to some degree, but in a rather arbitrary and unpredictable way.

4.4. Problems in the modelling of modal mass

Modelling of floor bays with identical simply supported rectangular plates simplifies the calculation of modal mass which must always be one-quarter of the total floor mass. However, recently developed finite element models [26], updated to match the modal properties measured on three full-scale in situ cast floor structures, showed that modal masses of real-life floors may be considerably less, sometimes as low as 5% for global modes of vibration.

4.5. Problems in modelling the effects of the higher modes of vibration

The concept of two families of mode shapes in two orthogonal directions which superimpose has little physical meaning. Although CSTR43 acknowledges that this assumption is conservative, it may easily become over-conservative due to double-counting of the same modes in the x- and y-directions.

4.6. Problems with limited application

The application of CSTR43 is limited to floors having a regular grid and geometry with perfect simply supported edge conditions, which typically do not exist in real-life situations. Further unwarranted approximations and considerable engineering judgement are required if the CSTR43 provision is to be applied to floors with a more irregular layout. This was the case with the floor structure shown in Fig. 3 and, clearly, one can have little confidence in response calculations made from such extrapolations.

5. Conclusions

Overall, the CSTR43 provision for checking the vibration serviceability of post-tensioned concrete floors is a very diligent attempt to produce a method where all calculations required can be made by hand. This underlying principle required a significant number of simplifications to be made, some conservative and others non-conservative. The consequences of these simplifications are that the final hand-calculated responses appear to be unreliable and arbitrary. Such response calculations are of little value despite the fact that they can be made by hand. Even if it is assumed that CSTR43 can provide a good order of magnitude check, the differences between the different vibration limits proposed in ISO 2631-2 are small (factor of 2) meaning that such crude checks are usually meaningless. To conclude, the principal failure of the CSTR43 vibration serviceability guidelines is that they over-simplify a very complex engineering problem, the solution of which is very sensitive to unwarranted assumptions. Therefore, it is proposed that Appendix G of the Concrete Society Technical Report 43 is withdrawn and more reliable design guidelines proposed for checking vibration serviceability of post-tensioned concrete floors.

Finally, as to a possible way forward, it is the authors' experience that even crude 3D linear dynamic finite element modelling of post-tensioned floors utilising standard orthotropic shell and beam elements, is likely to produce much more meaningful results than the CSTR43 guidelines. This modelling, however, should take into account the bending stiffness of the in situ supporting columns and walls and should pay due attention to typically irregular floor geometry [30]. Finite element modelling is therefore recommended for more critical applications where greater than usual slenderness of post-tensioned office floors is required.

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